

Consider the conic with rectangular equation $12x^2 - y^2 + 24x + 16y - 16 = 0$.

SCORE: ____ / 25 PTS

[a] Find the co-ordinates of the focus/foci.

$$\begin{aligned}12x^2 + 24x - y^2 + 16y &= 16 \\12(x^2 + 2x) - (y^2 - 16y) &= 16 \\12(x^2 + 2x + 1) - (y^2 - 16y + 64) &= 16 + 12 - 64 \quad (3) \\(2) \quad 12(x+1)^2 - (y-8)^2 &= -36 \quad (1) \\(3) \quad \frac{(y-8)^2}{36} - \frac{(x+1)^2}{3} &= 1\end{aligned}$$

$$c^2 = 36 + 3 = 39 \rightarrow c = \sqrt{39} \quad (3)$$

$$(-1, 8 \pm \sqrt{39}) \quad (2) \quad (2)$$

[b] If the equation corresponds to a circle, find its radius.

If the equation corresponds to a parabola, find its directrix.

If the equation corresponds to an ellipse, find the endpoints of its minor axis.

If the equation corresponds to a hyperbola, find the equations of its asymptotes.

$$y - 8 = \pm 2\sqrt{3}(x + 1) \quad (2) \quad (2) \quad (3)$$

Consider the polar equation $r = \frac{48}{5 - 7 \sin \theta}$.

SCORE: ____ / 25 PTS

[a] Find the equation of the directrix.

$$r = \frac{\frac{48}{5}}{1 - \frac{7}{5} \sin \theta} \quad (2)$$

$$e = \frac{7}{5} \quad (2)$$

$$ep = \frac{48}{5}$$

$$\frac{7}{5}p = \frac{48}{5} \quad (1)$$

$$p = \frac{48}{7}$$

$$y = -\frac{48}{7} \quad (2) \quad (2) \quad (2)$$

[b] Find the rectangular coordinates of the endpoints of all latera recta. Do NOT convert the equation to rectangular.

θ	r	(x, y)
0	$48/5$	$(48/5, 0)$
$\pi/2$	-24	$(0, -24)$
π	$48/5$	$(-48/5, 0)$
$3\pi/2$	4	$(0, -4)$

(4)

$$\text{CENTER} = (0, \frac{-24 + -4}{2}) = (0, -14) \quad (2)$$

$$\begin{aligned} \text{FOCI} &= (0, 2 * -14) \text{ AND } (0, 0) \\ &= (0, -28), (0, 0) \end{aligned}$$

$$\begin{aligned} \text{ENDS OF L.R.} &= (\pm 48/5, -28), (\pm 48/5, 0) \\ &\quad (2) \quad (2) \quad (2) \quad (2) \end{aligned}$$

Consider the polar equation $r = 2 - 5 \cos \theta$.

SCORE: ____ / 30 PTS

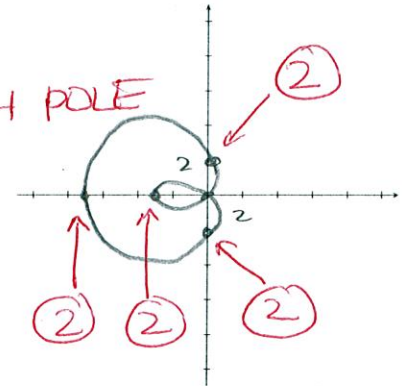
[a] Sketch the graph of the polar equation using the shortcut process shown in lecture / PowerPoint.

Label the scale on your axes clearly.

θ	r	(x, y)
0	-3	(-3, 0)
$\pi/2$	2	(0, 2)
π	7	(-7, 0)
$3\pi/2$	2	(0, -2)

⑤ SHAPE

② GOES THROUGH POLE



[b] Convert the equation to rectangular.

$$r^2 = 2r - 5r \cos \theta \quad ③$$

$$③ \quad x^2 + y^2 = 2\sqrt{x^2 + y^2} - 5x \quad ③$$

$$x^2 + y^2 + 5x = 2\sqrt{x^2 + y^2} \quad ③$$

$$(x^2 + y^2 + 5x)^2 = 4x^2 + 4y^2 \quad ③$$

OR

$$r = 2 - \frac{5x}{r} \quad ③$$

$$r^2 = 2r - 5x \quad ③$$

$$③ \quad x^2 + y^2 = 2\sqrt{x^2 + y^2} - 5x$$

$$x^2 + y^2 + 5x = ③ \quad 2\sqrt{x^2 + y^2}$$

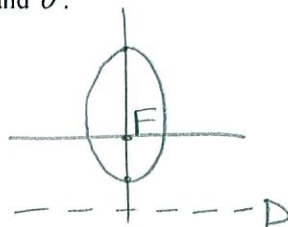
$$(x^2 + y^2 + 5x)^2 = 4x^2 + 4y^2 \quad ③$$

An ellipse has a focus at the pole and vertices with **rectangular** co-ordinates $(0, -5)$ and $(0, 8)$.

SCORE: ____ / 20 PTS

- [a] Give polar co-ordinates for the vertices, using positive values of r and θ .

$$\left(5, \frac{3\pi}{2}\right) \quad \left(8, \frac{\pi}{2}\right)$$



- [b] Find the **polar** equation of the ellipse.

$$r = \frac{ep}{1 - e \sin \theta}$$

$$5 = \frac{ep}{1+e} \quad 8 = \frac{ep}{1-e}$$

$$ep = 5 + 5e = 8 - 8e$$

$$13e = 3$$

$$e = \frac{3}{13}$$

$$\frac{3}{13}p = 5 + \frac{15}{13}$$

$$3p = 65 + 15$$

$$p = \frac{80}{3}$$

$$r = \frac{\frac{80}{13}}{1 - \frac{3}{13} \sin \theta}$$

$$r = \frac{80}{13 - 3 \sin \theta}$$

$$\uparrow$$

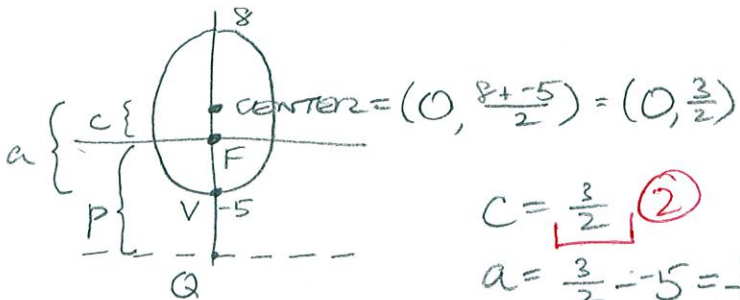
$$\textcircled{2}$$

$$\uparrow$$

$$\textcircled{2}$$

$\textcircled{2}$ SIMPLIFIED

[b] Find the polar equation of the ellipse.



$$c = \frac{3}{2} \text{ (2)}$$

$$a = \frac{3}{2} - (-5) = \frac{13}{2} \text{ (2)}$$

$$e = \frac{c}{a} = \frac{3}{13} \text{ (2)}$$

ALTERNATE
SOLUTION

$$e = \frac{VF}{VQ}$$

$$\frac{3}{13} = \frac{5}{p-5} \text{ (1)}$$

$$3p - 15 = 65$$

$$p = \frac{80}{3} \text{ (1)}$$

$$r = \frac{ep}{1 - e \sin \theta}$$

$$r = \frac{\frac{80}{13} \text{ (1)}}{1 - \frac{3}{13} \sin \theta \text{ (1)}}$$

$$r = \frac{80}{13 - 3 \sin \theta} \text{ (2)}$$

(2) SIMPLIFIED FINAL ANSWER

Name the shapes of the following graphs.

SCORE: ____ / 15 PTS

- [a] the locus of points in the plane that are twice as far from $(7, 7)$ as they are from $x = 7$

HYPERBOLA (3)

- [b] the locus of points in the plane that are 7 units closer to $(7, -7)$ than they are to $(7, 7)$

HYPERBOLA (3)

- [c] the graph with polar equation $r = 13 + 7 \cos \theta$

LIMACON WITH DIMPLE (2½)

- [d] the graph with polar equation $r = \frac{13}{7 - \sin \theta}$

ELLIPSE (2½)

- [e] the graph with polar equation $\theta = 5$

LINE (2)

- [f] the graph with equation $7 + 7x^2 - 13y + 13y^2 = 0$

ELLIPSE (2)

Consider the polar equation $r = 2 - \sin 2\theta$.

SCORE: ____ / 20 PTS

The following symmetry tests do NOT indicate that the graph is symmetric:

$(-r, -\theta)$, $(-r, \pi - \theta)$ and $(r, \pi - \theta)$

- [a] Using the results above, along with the tests and shortcuts shown in lecture, determine if the graph is symmetric over the polar axis, $\theta = \frac{\pi}{2}$ and/or the pole. Summarize your conclusions in the table on the right.

NOTE: Run as FEW tests as needed to prove your conclusions are correct.

POLAR AXIS: $(r, -\theta)$ $r = 2 - \sin 2(-\theta)$

$\textcircled{2} r = 2 + \sin 2\theta$

$\textcircled{2}$

POLE:

$(r, \theta) \rightarrow -r = 2 - \sin 2\theta$

$\textcircled{2} r = -2 + \sin 2\theta$

$\textcircled{2}$

$(r, \pi + \theta) r = 2 - \sin 2(\pi + \theta)$

$\textcircled{2} r = 2 - \sin(2\pi + 2\theta)$

$\textcircled{2} r = 2 - \sin 2\theta$

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION
Over the pole	SYMMETRIC

$\textcircled{3}$

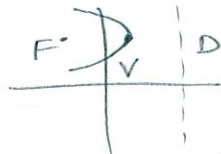
- [b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph)?

$\theta \in [0, \pi] \text{ or } [-\frac{\pi}{2}, \frac{\pi}{2}]$ $\textcircled{5}$

Find the rectangular equation of the parabola with focus $(-5, 7)$ and directrix $x = 13$.

SCORE: ____ / 15 PTS

$$\text{VERTEX} = \left(\frac{-5+13}{2}, 7 \right) = \underline{(4, 7)} \textcircled{3}$$



P = DIRECTED DISTANCE FROM VERTEX TO FOCUS = $-5 - 4 = -9$

$$\textcircled{4} (y-7)^2 = \textcircled{4} (-9) \textcircled{3} (x-4)$$

$$\textcircled{1} (y-7)^2 = -36 \textcircled{1} (x-4)$$